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Scheme for Probabilistic Remotely Preparing a *d*-Dimensional Equatorial Quantum State

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Abstract We present a scheme for probabilistic remote preparation of a d-dimensional equatorial quantum state. In the scheme, a bipartite d-dimensional partial entangled state is used as the quantum channel, and the single-qudit projective measurement and appropriate unitary transformation are needed. As a special, the remote preparation in three dimension is studied.

Keywords Remote state preparation · d-Dimensional quantum state · Generalized projective measurement

1 Introduction

Quantum entanglement has generated much interest in quantum information processing such as quantum teleportation [2], quantum telecloning [16], superdense coding [12], quantum cryptography [9], quantum computation [6] and so on. Quantum teleportation, proposed by Bennett et al. [2], is the process that transmits an unknown two-state particle, or a qubit from a sender (Alice) to a receiver (Bob) via a quantum channel with the help of some classical information. Several theoretical and experimental researches have extensive explored teleportation from all aspects [21]. Considering that an entangled state may be non-maximally entangled, Li et al. [11] proposed a non-maximally entangled quantum channel, which indicates that teleportation of a single-particle state can be realized with a certain probability. Based on work of Li et al. [11], some schemes of quantum probabilistic teleportation of three dimensional and d-dimensional states by using the partial entangled states as quantum channels were presented [1, 8, 17, 24].

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Another important application of the entanglement, which correlates closely to teleportation, is remote state preparation (RSP) [3, 15, 18]. RSP is called "teleportation of a know state". In RSP, Alice performs a measurement on her share of the entangled resource in a basis chosen in accordance with the state she wishes to help Bob in his laboratory to prepare. Recently, there have been many theoretical and experimental protocols for generalization of RSP [4, 5, 7, 10, 19, 20, 22, 23]. More recently, Liu et al. [13, 14] presented two schemes for remotely preparing d-dimensional equatorial entangled states. But they considered only the maximally entangled quantum channel. In the present paper, a scheme for probabilistic remotely preparing a d-dimensional equatorial quantum state is proposed. In this scheme, a bipartite d-dimensional non-maximally entangled state is employed as the quantum channel.

2 Remote Preparation of a Three-Dimensional Equatorial Quantum State

To present our scheme more clearly, let us first begin with remote state preparation in three dimensions. Suppose that Alice wishes to help Bob remotely prepare a three-dimension equatorial quantum state

$$|\phi\rangle = \frac{1}{\sqrt{3}}(|0\rangle + e^{i\eta_1}|1\rangle + e^{i\eta_2}|2\rangle), \tag{1}$$

where the parameters η_1 and η_2 are real. This state is known completely state to Alice but unknown to Bob. Without losing generality, we suppose that the bipartite three-dimension maximal entangled state $|\varphi\rangle_{12}$ shared by Alice and Bob can be written as

$$|\varphi\rangle_{12} = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)_{12}, \tag{2}$$

where qutrit 1 belongs to Alice, and qutrit 2 belongs to Bob. In order to help Bob construct the state (1), what Alice needs to do is to make a projective measurement on her qutrit 1. The measurement basis chosen by Alice is a set of mutually orthogonal basis vectors $\{|x_0\rangle, |x_1\rangle, |x_2\rangle\}$, which are related to computation basis vectors $\{|0\rangle, |1\rangle, |2\rangle\}$ is given by

$$|x_k\rangle_1 = \frac{1}{\sqrt{3}}\sum_j e^{2\pi ijk/3}e^{-i\eta_j}|j\rangle_1, \tag{3}$$

where $\eta_0 = 0$ and $j, k = 0, 1, 2$. Specifically,

$$\begin{aligned}
|x_0\rangle_1 &= \frac{1}{\sqrt{3}}(|0\rangle_1 + e^{-i\eta_1}|1\rangle_1 + e^{-i\eta_2}|2\rangle_1),\\
|x_1\rangle_1 &= \frac{1}{\sqrt{3}}(|0\rangle_1 + e^{2\pi i/3}e^{-i\eta_1}|1\rangle_1 + e^{4\pi i/3}e^{-i\eta_2}|2\rangle_1),\\
|x_2\rangle_1 &= \frac{1}{\sqrt{3}}(|0\rangle_1 + e^{4\pi i/3}e^{-i\eta_1}|1\rangle_1 + e^{2\pi i/3}e^{-i\eta_2}|2\rangle_1).
\end{aligned} \tag{4}$$

Then, we have

$$\begin{aligned}
|\varphi\rangle_{12} &= \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle)\\
&= \frac{1}{\sqrt{3}}\sum_k |x_k\rangle_1 \otimes |\psi_k\rangle_2
\end{aligned} \tag{5}$$

with $|\psi_k\rangle_2=\sum_j e^{-2\pi ijk/3}e^{i\eta_j}|j\rangle_2/\sqrt{3}$. More Specifically,

$$\begin{aligned}|\psi_0\rangle_2&=\frac{1}{\sqrt{3}}(|0\rangle_2+e^{i\eta_1}|1\rangle_2+e^{i\eta_2}|2\rangle_2),\\|\psi_1\rangle_2&=\frac{1}{\sqrt{3}}(|0\rangle_2+e^{4\pi i/3}e^{i\eta_1}|1\rangle_2+e^{2\pi i/3}e^{i\eta_2}|2\rangle_2),\\|\psi_2\rangle_2&=\frac{1}{\sqrt{3}}(|0\rangle_2+e^{2\pi i/3}e^{i\eta_1}|1\rangle_2+e^{4\pi i/3}e^{i\eta_2}|2\rangle_2).\end{aligned}\tag{6}$$

Now let Alice measures her qutrit 1. From (5), we find that if the result of Alice's measurement is $|x_k\rangle_1$ and she informs Bob of her measurement outcome via a classical channel, the qutrit 2 at Bob's side will collapse into the state $|\psi_k\rangle_2$. Obviously if $k=0$, the state $|\psi_0\rangle_2$ is identical to the original state $|\phi\rangle$ that Alice wishes to prepare remotely. If $k=1$ or $k=2$, Bob needs to perform a local unitary transformation U_k on qutrit 2 so that the state $|\psi_k\rangle_2$ can be turn into his desired result $|\psi_0\rangle_2$,

$$U_k|\psi_k\rangle_2=|\psi_0\rangle_2,\tag{7}$$

where $U_k=\sum_j e^{2\pi ijk/3}|j\rangle\langle j|$. Explicitly,

$$U_0=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix},\qquad U_1=\begin{pmatrix}1&0&0\\0&e^{2\pi i/3}&0\\0&0&e^{4\pi i/3}\end{pmatrix},\qquad U_2=\begin{pmatrix}1&0&0\\0&e^{4\pi i/3}&0\\0&0&e^{2\pi i/3}\end{pmatrix}.\tag{8}$$

In this way, the RSP scheme in three dimension can be successfully realized.

Now let us turn to another case. Suppose that Alice wishes to prepare remotely is still the state (1), but the entangled state shared by Alice and Bob is a non-maximally entangled state, which is given by

$$|\psi\rangle_{12}=a_0|00\rangle_{12}+a_1|11\rangle_{12}+a_2|22\rangle_{12},\tag{9}$$

where $|a_0|^2+|a_1|^2+|a_2|^2=1$ and $|a_2|<|a_0|$, $|a_2|<|a_1|$. We assume qutrit 1 belongs to Alice while qutrit 2 to Bob. Alice carries out a single-qutrit projective measurement on the qutrit 1 in a set of mutually orthogonal basis vectors $\{|x_0\rangle,|x_1\rangle,|x_2\rangle\}$. The state $|\psi\rangle_{12}$ can be rewritten as

$$\begin{aligned}|\psi\rangle_{12}=\frac{1}{\sqrt{3}}[&|x_0\rangle_1(a_0|0\rangle_2+a_1e^{i\eta_1}|1\rangle_2+a_2e^{i\eta_2}|2\rangle_2)\\&+|x_1\rangle_1(a_0|0\rangle_2+a_1e^{4\pi i/3}e^{i\eta_1}|1\rangle_2+a_2e^{2\pi i/3}e^{i\eta_2}|2\rangle_2)\\&+|x_2\rangle_1(a_0|0\rangle_2+a_1e^{2\pi i/3}e^{i\eta_1}|1\rangle_2+a_2e^{4\pi i/3}e^{i\eta_2}|2\rangle_2)].\end{aligned}\tag{10}$$

After her measurement, Alice transmits her measurement result to Bob. According to the result Alice's measurement Bob tries to reconstruct the original state on his qutrit 2. As proposed above, if the result of Alice measurement is $|x_k\rangle_1$ $(k=0,1,2)$, Bob should operate a unitary transformation $U_k=\sum_j e^{2\pi ijk/3}|j\rangle\langle j|$ $(j,k=0,1,2)$ on his qutrit 2, respectively. Thus, the state of qutrit 2 can evolves to $\frac{1}{\sqrt{3}}(a_0|0\rangle_2+a_1e^{i\eta_1}|1\rangle_2+a_2e^{i\eta_2}|2\rangle_2)$. Next, Bob introduces an auxiliary qutrit A in the state $|0\rangle_A$, and performs another unitary transformation $U_k^{'}$ on both qutrit 2 and A. $U_k^{'}$ is a 9×9 matrix in the two-qutrit bases $\{|00\rangle_{2A}, |10\rangle_{2A}, |20\rangle_{2A}, |01\rangle_{2A}, |11\rangle_{2A}, |21\rangle_{2A}, |02\rangle_{2A}, |12\rangle_{2A}, |22\rangle_{2A}\}$

$$U_k^{'} = \begin{pmatrix} \omega_0 & \omega_1 & 0 \\ \omega_1 & -\omega_0 & 0 \\ 0 & 0 & I \end{pmatrix}, \tag{11}$$

where I is a 3×3 identity matrix, ω_0 and ω_1 are 3×3 diagonal matrices. ω_0 and ω_1 take the forms as follows

$$\omega_0 = \mathrm{diag}(\lambda_0, \lambda_1, 1), \tag{12}$$

$$\omega_1 = \mathrm{diag}\Big(\sqrt{1-\lambda_0^2}, \sqrt{1-\lambda_1^2}, 0\Big), \tag{13}$$

where $\lambda_0 = a_2/a_0$, and $\lambda_1 = a_2/a_1$. The unitary transformation $U_k^{'}$ will transform the state $\frac{1}{\sqrt{3}}(a_0|0\rangle_2 + a_1 e^{i\eta_1}|1\rangle_2 + a_2 e^{i\eta_2}|2\rangle_2) \otimes |0\rangle_A$ into

$$\frac{a_2}{\sqrt{3}}(|0\rangle_2 + e^{i\eta_1}|1\rangle_2 + e^{i\eta_2}|2\rangle_2) \otimes |0\rangle_A + \frac{1}{\sqrt{3}}\Big(\sqrt{a_0^2 - a_2^2}|0\rangle_2 + \sqrt{a_1^2 - a_2^2}e^{i\eta_1}|1\rangle_2\Big) \otimes |1\rangle_A. \tag{14}$$

Then Bob performs a single-qutrit measurement on the auxiliary qutrit A. If the measurement result is $|1\rangle_A$ or $|2\rangle_A$, the remote preparation of the original state fails. If the measure outcome is $|0\rangle_A$, the state of the qutrit 2 collapses to $(|0\rangle + e^{i\eta_1}|1\rangle + e^{i\eta_2}|2\rangle)/\sqrt{3}$, which is exactly the original state $|\phi\rangle$.Thus the remote preparation of single qutrit state is successfully realized and the success probability is $|a_2|^2$.

3 Probabilistic Remote Preparation of a d-Dimensional Equatorial Quantum State

Now, we will generalize the above scheme to arbitrary higher dimension case. Suppose that Alice wishes to help Bob remotely prepare an unknown qudit state

$$|\phi\rangle = \sum_{j=0}^{d-1} e^{i\eta_j}|j\rangle/\sqrt{d}, \tag{15}$$

where the parameter η_j is real and $\eta_0 = 0$. We assume the state (15) is known completely to Alice but unknown to Bob. We also suppose that a bipartite d-dimensional nonmaximally entangled state shared by Alice and Bob can be written as

$$|\psi\rangle_{12} = \sum_{j=0}^{d-1} a_j|jj\rangle_{12}/\sqrt{d}, \tag{16}$$

where $\sum_{j=0}^{d-1}|a_j|^2 = 1$, $|a_{d-1}| < |a_j|$ $(j = 0, 1, \ldots, d-2)$, and the qudit 1 belongs to Alice and the qudit 2 to Bob. To achieve the RSP scheme, Alice needs to do is to make a projective measurement on her qudit 1. The measurement basis chosen by Alice is a set of mutually orthogonal vectors $\{|x_0\rangle, |x_1\rangle, |x_2\rangle, \ldots, |x_{d-1}\rangle\}$, which are related to computation basis vectors $\{|0\rangle, |1\rangle, |2\rangle, \ldots, |d-1\rangle\}$ is given by

$$|x_k\rangle = \sum_{j=0}^{d-1} e^{2\pi ikj/d} e^{-i\eta_j}|j\rangle/\sqrt{d} \quad (k, j = 0, 1, 2, \ldots, d-1). \tag{17}$$

Let Alice carries out a single-qudit projective measurement on the qudit 1 in the orthogonal basis vectors $\{|x_0\rangle, |x_1\rangle, |x_2\rangle, \ldots, |x_{d-1}\rangle\}$. The state $|\psi\rangle_{12}$ can be rewritten as

$$|\psi\rangle_{12} = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} \left[|x_k\rangle_1 \sum_{j=0}^{d-1} a_j e^{i\eta_j} |j\rangle_2 \right]. \tag{18}$$

After Alice makes the single-qudit measurement, she informs Bob of the measurement result, then Bob reconstruct the original state on his qudit 2. As proposed above, the result of Alice measurement is $|x_k\rangle_1$ ($k = 0, 1, 2, \ldots, d-1$), Bob should operate a unitary transformation $U_d = \sum_j e^{2\pi i jk/d} |j\rangle\langle j|$ ($j, k = 0, 1, 2, \ldots, d-1$), respectively. In this case, the system state evolves to $\frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} a_j e^{i\eta_j} |j\rangle_2$. Next, Bob introduces an auxiliary qutrit A in the state $|0\rangle_A$, and then performs another unitary transformation $U_d^{'}$ on both qudit 2 and A. $U_d^{'}$ is a $3d \times 3d$ matrix in the basis $\{|00\rangle_{2A}, |10\rangle_{2A}, \ldots, |(d-1)0\rangle_{2A}, |01\rangle_{2A}, |11\rangle_{2A}, \ldots, |(d-1)1\rangle_{2A}, |02\rangle_{2A}, |12\rangle_{2A}, \ldots, |(d-1)2\rangle_{2A}\}$

$$U_d^{'} = \begin{pmatrix} \upsilon_0 & \upsilon_1 & 0 \\ \upsilon_1 & -\upsilon_0 & 0 \\ 0 & 0 & I \end{pmatrix}, \tag{19}$$

where I is a $d \times d$ identity matrix, υ_0 and υ_1 are $d \times d$ diagonal matrices. υ_0 and υ_1 take the forms as follows

$$\upsilon_0 = \mathrm{diag}(\lambda_0, \lambda_1, \ldots, \lambda_{d-2}, 1), \tag{20}$$

$$\upsilon_1 = \mathrm{diag}(\sqrt{1-\lambda_0^2}, \sqrt{1-\lambda_1^2}, \ldots, \sqrt{1-\lambda_{d-2}^2}, 0), \tag{21}$$

where $\lambda_j = a_{d-1}/a_j$ $(j = 0, 1, \ldots, d-2)$. After the transformation $\frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} a_j e^{i\eta_j} |j\rangle_2$ evolves to

$$\frac{a_{d-1}}{\sqrt{d}} \left(\sum_{j=0}^{d-1} e^{i\eta_j} |j\rangle_2 \right) \otimes |0\rangle_A + \frac{1}{\sqrt{3}} \left(\sum_{j=0}^{d-2} \sqrt{a_j^2 - a_{d-1}^2} e^{i\eta_j} |j\rangle_2 \right) \otimes |1\rangle_A. \tag{22}$$

Then Bob performs a single-qutrit measurement on the auxiliary qutrit A. If the measurement result is $|1\rangle_A$ or $|2\rangle_A$, the remote preparation of the original state fails. If the measure outcome is $|0\rangle_A$, the state of the qudit 2 collapses to the state $\sum_{j=0}^{d-1} e^{i\eta_j} |j\rangle/\sqrt{d}$. That is, the remote preparation of the original state $|\phi\rangle$ is successfully realized with the success probability being $|a_{d-1}|^2$.

4 Conclusion

We proposed a scheme for probabilistic RSP of a d-dimensional equatorial quantum state. To present the scheme more clearly, we first investigate a scheme of RSP of a three-dimension equatorial quantum state by using a bipartite three-dimension partial entangled state as quantum channel. Then we directly generalize the scheme to d-dimensional equatorial case. It is shown that, for the bipartite d-dimensional nonmaximally entangled state shared by Alice and Bob as quantum channel, the probabilistic RSP of a d-dimensional equatorial quantum state can be successfully realized with the single-qudit projective measurement and appropriate unitary transformation. It is also shown that the success probability of RSP is determined only by the smallest coefficient of the bipartite entangled state that is used as quantum channel.

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